Fusion barrier distributions in systems with finite excitation energy

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Abstract

Eigen-channel approach to heavy-ion fusion reactions is exact only when the excitation energy of the intrinsic motion is zero. In order to take into account effects of finite excitation energy, we introduce an energy dependence to weight factors in the eigen-channel approximation. Using two channel problem, we show that the weight factors are slowly changing functions of incident energy. This suggests that the concept of the fusion barrier distribution still holds to a good approximation even when the excitation energy of the intrinsic motion is finite. A transition to the adiabatic tunneling, where the coupling leads to a static potential renormalization, is also discussed.

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It has been well recognized that cross sections of heavy-ion fusion reactions at energies near and below the Coulomb barrier are strongly influenced by coupling of the relative motion of the colliding nuclei to nuclear intrinsic motions[1]. When the intrinsic degree of freedom has a degenerate spectrum, fusion cross sections can be calculated in the sudden approximation. In this limit, the coupling gives rise to a distribution of fusion potentials. The fusion cross section is then given by an average over the contributions from each fusion barrier with appropriate weight factors[2, 3, 4]. Based on this idea, a method was proposed to extract the barrier distribution directly from the fusion excitation function using the second derivative with respect to the product of the fusion cross section and the center of mass energy, $E\sigma$ [5]. This stimulated precise measurements of the fusion cross sections for several systems[6]. The extracted barrier distributions have then been found to be very sensitive to the structure of the colliding nuclei, e.g. the sign of hexadecapole deformation parameter, while the fusion cross section itself is rather featureless[6, 7].

In a rigorous theoretical interpretation, the barrier distribution representation, i.e. the second derivative of $E\sigma$, has a clear physical meaning only if the excitation energy of the intrinsic motion is zero. Nonetheless, this analysis has been successfully applied to systems with relatively large excitation energies[6, 8, 9]. For example, the second derivative of $E\sigma$ for ^{16}O + ^{144}Sm fusion reaction clearly shows the effects of coupling to the octupole phonon state in ^{144}Sm, whose excitation energy is 1.8 MeV, much more clearly than the fusion cross section itself[6, 8]. Also the analysis of the fusion reaction between ^{58}Ni and ^{60}Ni, where the excitation energies of quadrupole phonon states are 1.45 and 1.33 MeV, respectively, shows that the barrier distribution representation depends strongly on the number of phonon states included in coupled-channels calculations[9]. These analyses suggest that the representation of fusion process in terms of the second derivative of $E\sigma$ is a very powerful method to study the details of the effects of nuclear structure, irrespective of the excitation energy of the intrinsic motion.

Despite these successes, there remains the question whether the second derivative of $E\sigma$ represents a "distribution "of fusion potential barriers when the excitation energy of the intrinsic motion is not zero. To address this problem, one may attempt to use the constant coupling approximation, where the coupling form factor is assumed to be constant throughout the interaction range [10]. In this approximation, the unitary transformation gives the fusion cross section as a sum of eigen-channel cross sections. The constant coupling approximation has, however, a serious drawback in that the results strongly depend on the position where the strength of the coupling Hamiltonian is evaluated. Therefore, one needs to use a different approach to address the question.

In this paper we treat the weight factors as energy dependent variables. The possibility of the energy dependence of weight factors when the excitation energy is finite has been suggested in ref. [11]. Here, we explicitly study the energy dependence by performing exact coupled-channels calculations. It will be shown that the energy dependence is quite weak for a wide range of excitation energy, suggesting that the eigen-channel approximation works well even when the excitation energy of the intrinsic motion is not small.

Let us consider, for example, the case where the intrinsic degree of freedom has only

two levels. The coupled-channels equations then read

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + U(R) + \begin{pmatrix} 0 & F(R) \\ F(R) & \epsilon \end{pmatrix}\right] \begin{pmatrix} u_0(R) \\ u_1(R) \end{pmatrix} = E \begin{pmatrix} u_0(R) \\ u_1(R) \end{pmatrix}, \tag{1}$$

where $U(R)$ and μ are the bare potential and the reduced mass of the relative motion, respectively. $F(R)$ is the coupling form factor and ϵ is the excitation energy of the intrinsic motion, respectively. When the excitation energy ϵ is zero, the unitary matrix which diagonalizes the coupling matrix is independent of the position R, and the barrier penetrability is exactly given by

$$P(E) = \frac{1}{2}\left[P_0(E; U(R) + F(R)) + P_0(E; U(R) - F(R))\right], \tag{2}$$

where $P_0(E; V(R))$ is the penetrability of potential $V(R)$ at energy E. In this limit, the original single barrier splits into two effective potential barrier with equal weights.

If the excitation energy ϵ is not zero the unitary matrix depends explicitly on the position R. The unitary matrix, therefore, does not commute with the kinetic energy operator, and hence simple eigne-channel approach does not hold. An approximation which is often made is to assume that the coupling form factor $F(R)$ is a constant. In this approximation, the unitary matrix becomes independent of R, and the penetrability is given by a formula similar to eq. (2) but with different eigen-values and weight factors [10]. In order to take into account the radial dependence of the coupling form factor, an approximation was proposed, where the coupling matrix is diagonalized to obtain the eigen-barriers at each position of the relative motion R while the weight factors are calculated at a chosen position R_w [13]. The computer code CCMOD uses this approximation[14].

Even when we introduce this approximation, the weight factors are still functions of the chosen position R_w, and the results might strongly depend on that choice. Usually R_w is chosen to be the position of maximum of the bare potential barrier[13, 14]. However, there is no theoretical justification that this is the optimum choice, and furthermore, it is not obvious whether one can determine the weight factors independent of the incident energy.

In order to avoid these drawbacks and examine the energy dependence of the weight factors, we parametrize the penetrability as

$$P(E) = v_+(E)P_+(E) + v_-(E)P_-(E), \tag{3}$$

and evaluate the weight factors $v_\pm$ at each incident energy E. Here $P_\pm(E)$ are the penetrabilities of the eigen-potentials $U(R)+\lambda_\pm(R)$, respectively, $\lambda_\pm(R)$ being the eigen-values of the coupling matrix at each position of R, which are given by

$$\lambda_\pm(R) = \left(\epsilon \pm \sqrt{\epsilon^2 + 4F(R)^2}\right)/2. \tag{4}$$

If the weight factors slowly vary as functions of the incident energy, the effects of channel coupling can be interpreted in terms of the barrier *distribution*, even when the excitation energy is non-zero. In the appendix, we generalize eq. (3) to multi-channel case using the

path integral approach. Since the weight factors have to satisfy the unitarity condition[3], they can be uniquely determined in the present two level problem, and are given by

$$v_+(E) = (P(E) - P_-(E))/(P_+(E) - P_-(E)), \quad v_-(E) = (P_+(E) - P(E))/(P_+(E) - P_-(E)). \tag{5}$$

We performed coupled-channels calculations to examine the energy dependence of the weight factors. To this end, the coupled-channels equations have to be solved with good accuracy, since the penetrabilities $P(E)$ and $P_\pm(E)$ in eq. (5) are exponentially small quantities at energies below the barrier. The incoming wave boundary condition, which is often used in coupled channels calculations for heavy-ion collisions [15], can bring some numerical errors, though they would be small enough for the purpose of calculating fusion cross sections. In order to avoid this, we use here a schematic model of heavy-ion fusion reactions by Dasso *et al.*[10], where the radial motion between colliding nuclei is treated as a one dimensional barrier penetration problem. Following ref. [10], we assume the gaussian shape for both the bare potential and the coupling form factor,

$$U(R) = U_0 e^{-R^2/2s^2}, \quad F(R) = F_0 e^{-R^2/2s_f^2}. \tag{6}$$

The parameters are chosen following ref. [10] to be $U_0 = 100$ MeV, $F_0 = 3$ MeV, and $s = s_f = 3$ fm, respectively, which mimic the fusion reaction between two ^{58}Ni nuclei. We have checked that our conclusions do not significantly change as long as the value of s_f is not too small. The mass μ and the excitation energy ϵ are taken to be 29 m_N, m_N being the nucleon mass, and 2 MeV, respectively.

The upper panel of Fig. 1 shows potential barriers for the present problem. The dotted line is the bare potential $U(R)$ and the solid and the dashed lines are the eigen-potentials $U(R) + \lambda_-(R)$ and $U(R) + \lambda_+(R)$, respectively. If we adopt the eigen-channel picture, Fig. 1 means that the potential barriers are "distributed "and the original bare potential (the dotted line) splits into the two potentials (the solid and the dashed lines). Because of the non-commutatibity of the unitary matrix which diagonalizes the coupling matrix, and the kinetic energy operator, the standard eigen-channel picture does not apply. If we ignore this non-commutatibity (i.e., if we adopt with the CCMOD prescription) the weight factors are given by

$$w_\pm(R) = F(R)^2 / \left(F(R)^2 + \lambda_\pm(R)^2\right). \tag{7}$$

They are shown in the lower panel of Fig. 1 as a function of the distance R. Although they do not vary so much near the barrier position, their changes are appreciable throughout the barrier region. Therefore, the prescription to fix the weight factors to the values at the barrier position may not be satisfactory.

The results of exact coupled-channels calculations are shown in Fig. 2. The first and the second panels are the penetrabilities and their first derivative with respect to the energy, respectively. The latter corresponds to the second derivative of $E\sigma$ [5]. We use the point difference formula with $\Delta E = 2$ MeV to obtain the first derivative of the penetrability, as is often done in the analyses of heavy-ion fusion reactions[6]. The first derivative of the penetrability has a clear double-peaked structure, which could be

associated with the two eigen-potential barriers and could thus be interpreted in terms of a 'barrier distribution '. In order to see whether this is the case, we plot in the last panel of Fig. 2 the optimum weight factors defined by eq. (5). The solid and the dashed lines in the figure correspond to the weight factors for the lower and the higher potentials, respectively. We observe that the optimum weight factors change only slightly as functions of the incident energy; their change is barely 2.6 % from 10 MeV below the barrier to 10 MeV above the barrier. Another important result of this calculation is that the weight factors are considerably different from those estimated in the usual way, i.e. at the barrier position. When the constant coupling approximation was first introduced, it was expected that the choice of the position where the weight factors are estimated is not critical to calculate tunneling probabilities because the weight factors vary slowly across the barrier region[10, 13]. The position was then chosen to be the barrier position of the uncoupled barrier. Contrary to this expectation, our calculations show that determining the weight factors at the barrier position does not give proper weight factors even though the weight factors do not have a strong radial dependence. The situation would be much more serious in realistic calculations, since the weights are still changing at the barrier position due to the fact that the coupling extends outside the Coulomb barrier[11].

The same calculations were repeated for different values of the excitation energy ϵ and the results are shown in Fig. 3. The quantities shown in each panel are the same as those in Fig. 2 except for the third panel, where only the optimum weight factors for the lower barrier are plotted. We again observe that the weight factor changes only marginally as a function of the incident energy, even when the excitation energy is finite. We thus conclude that the eigen-channel approach is still applicable even when the excitation energy of the intrinsic motion is finite.

When the excitation energy ϵ is zero, the barrier distribution, i.e. the first derivative of the penetrability has two symmetric peaks and the weight factor has no incident energy dependence (the solid line). As the excitation energy increases, some strength is transfered from the higher peak to the lower peak, and the barrier distribution becomes asymmetric. As the excitation energy significantly exceeds the curvature of the bare barrier, which is about 4 MeV in our example, one expects to reach the adiabatic limit. To illustrate this, figure 4 shows the influence of the coupling to an excited state whose energy is 8 MeV. The figure also contains the result for the the no coupling case (the dotted line) for comparison. In this case, the weight for the higher peak is considerably smaller than that for the lower peak, and the barrier distribution has essentially only a single peak. The peak position, however, is shifted towards a lower energy. This is consistent with the adiabatic picture, i.e. the main effect of the coupling to a state whose excitation energy is much larger the barrier curvature is to introduce an energy independent shift of the potential accompanied with the mass renormalization[12, 16]. Correspondingly, the barrier distribution is only shifted without significantly changing its shape unless the coupling form factor has a strong radial dependence[17]. It is thus clear that the second derivative of $E\sigma$ can represent a wide range of situations between the two extreme limits, i.e. from the adiabatic limit where the coupling leads to an adiabatic potential renormalization, to the sudden limit where the coupling gives rise to a barrier distribution.

In summary, we have discussed the energy dependence of the weight factors in the eigen

channels approximation. Performing exact coupled-channels calculations, we showed that the energy dependence is very weak, regardless of the excitation energy of the intrinsic motion. Therefore, it makes sense to call the second derivative of $E\sigma$ as the fusion barrier distribution, even when the excitation energy is finite. We also discussed a transition from the sudden to the adiabatic tunneling limits, and showed that the fusion barrier distribution can represent these two limits in a natural way.

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APPENDIX: EIGEN-CHANNEL APPROXIMATION IN PATH INTEGRAL APPROACH

Consider a system where a macroscopic degree of freedom R couples to an intrinsic degree of freedom ξ. We assume the following Hamiltonian for this system;

$$H(R,\xi) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial R^2} + U(R) + H_0(\xi) + V(R,\xi), \tag{8}$$

where μ is the mass for the macroscopic motion, $U(R)$ the potential in the absence of the coupling, $H_0(\xi)$ the Hamiltonian for the internal motion, and $V(R,\xi)$ the coupling between them. The barrier transmission probability from the initial position R_i on the right side of the barrier to the final position R_f on the left side is then given by [18],

$$\begin{aligned} P(E) &= \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_i P_f}{\mu^2}\right) \int_0^\infty dT\ e^{(i/\hbar)ET} \int_0^\infty d\widetilde{T}\ e^{-(i/\hbar)E\widetilde{T}} \\ &\times \int \mathcal{D}[R(t)] \int \mathcal{D}[\widetilde{R}(\tilde{t})] e^{(i/\hbar)[S_t(R,T)-S_t(\widetilde{R},\widetilde{T})]} \rho_M(\widetilde{R}(\tilde{t}),\widetilde{T};R(t),T), \end{aligned} \tag{9}$$

where E is the total energy of the system, and P_i and P_f are the classical momenta at R_i and R_f, respectively. $S_t(R,T)$ is the action for the macroscopic motion along a path $R(t)$, and is given by

$$S_t(R,T) = \int_0^T dt \left(\frac{1}{2}\mu\dot{R}(t)^2 - U(R(t))\right). \tag{10}$$

The effects of the internal degree of freedom are included in the two time influence functional ρ_M, which is defined as

$$\rho_M(\widetilde{R}(\tilde{t}),\widetilde{T};R(t),T) = \sum_{n_f} < n_i|\hat{u}^\dagger(\widetilde{R}(\tilde{t}),\widetilde{T})|n_f >< n_f|\hat{u}(R(t),T)|n_i > . \tag{11}$$

Here, n_i and n_f are the initial and the final states of the internal motion at position R_i and R_f, respectively. The time evolution operator $\hat{u}(R(t),t)$ of the intrinsic motion obeys

$$i\hbar\frac{\partial}{\partial t}\hat{u}(R(t),t) = [H_0(\xi) + V(R(t),\xi)]\hat{u}(R(t),t). \tag{12}$$

with the initial condition $\hat{u}(R, t=0) = 1$.

We introduce the eigen-channel (or the adiabatic) basis, as

$$(H_0(\xi) + V(R,\xi))\varphi_n(R,\xi) = \lambda_n(R)\varphi_n(R,\xi). \tag{13}$$

Expanding the intrinsic wave function at time t in this basis

$$\hat{u}(R,t)|n_i> = \sum_n a_n(t)\exp\left[-\frac{i}{\hbar}\int_0^t dt'\lambda_n\left(R(t')\right)\right]|\varphi_n(R(t))>, \tag{14}$$

Eq. (2) for the barrier penetrability reads

$$\begin{aligned} P(E) &= \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}}\left(\frac{P_iP_f}{\mu^2}\right)\int_0^\infty dT\ e^{(i/\hbar)ET}\int_0^\infty d\widetilde{T}\ e^{-(i/\hbar)E\widetilde{T}} \\ &\times \int\mathcal{D}[R(t)]\int\mathcal{D}[\widetilde{R}(\tilde{t})]\sum_{n,m}a_n(T)a_m(\widetilde{T})^* < \varphi_m(\widetilde{R}(\widetilde{T}))|\varphi_n(R(T))> \\ &\times e^{\frac{i}{\hbar}\int_0^T dt\left(\frac{1}{2}\mu\dot{R}^2 - U(R) - \lambda_n(R)\right)}e^{-\frac{i}{\hbar}\int_0^{\widetilde{T}} d\tilde{t}\left(\frac{1}{2}\mu\dot{\widetilde{R}}^2 - U(\widetilde{R}) - \lambda_m(\widetilde{R})\right)}. \end{aligned} \tag{15}$$

We use the semi-classical approximation, and for energies well below the barrier, where the single paht dominates, evaluate the path integral along the classical path

$$R(t) = \widetilde{R}(\tilde{t}) = R_{cl}(t), \qquad T = \widetilde{T}^* = T_{cl}. \tag{16}$$

In this case, the orthogonalitiy of the adiabatic basis leads to

$$P(E) = \sum_n v_n(E)P_0(E; U(R) + \lambda_n(R)), \tag{17}$$

where the weight factors are given by

$$v_n(E) = |a_n(T_{cl})|^2. \tag{18}$$

The weight factors depend implicitly on the energy E through the time evolution of the intrinsic system along the classical path.

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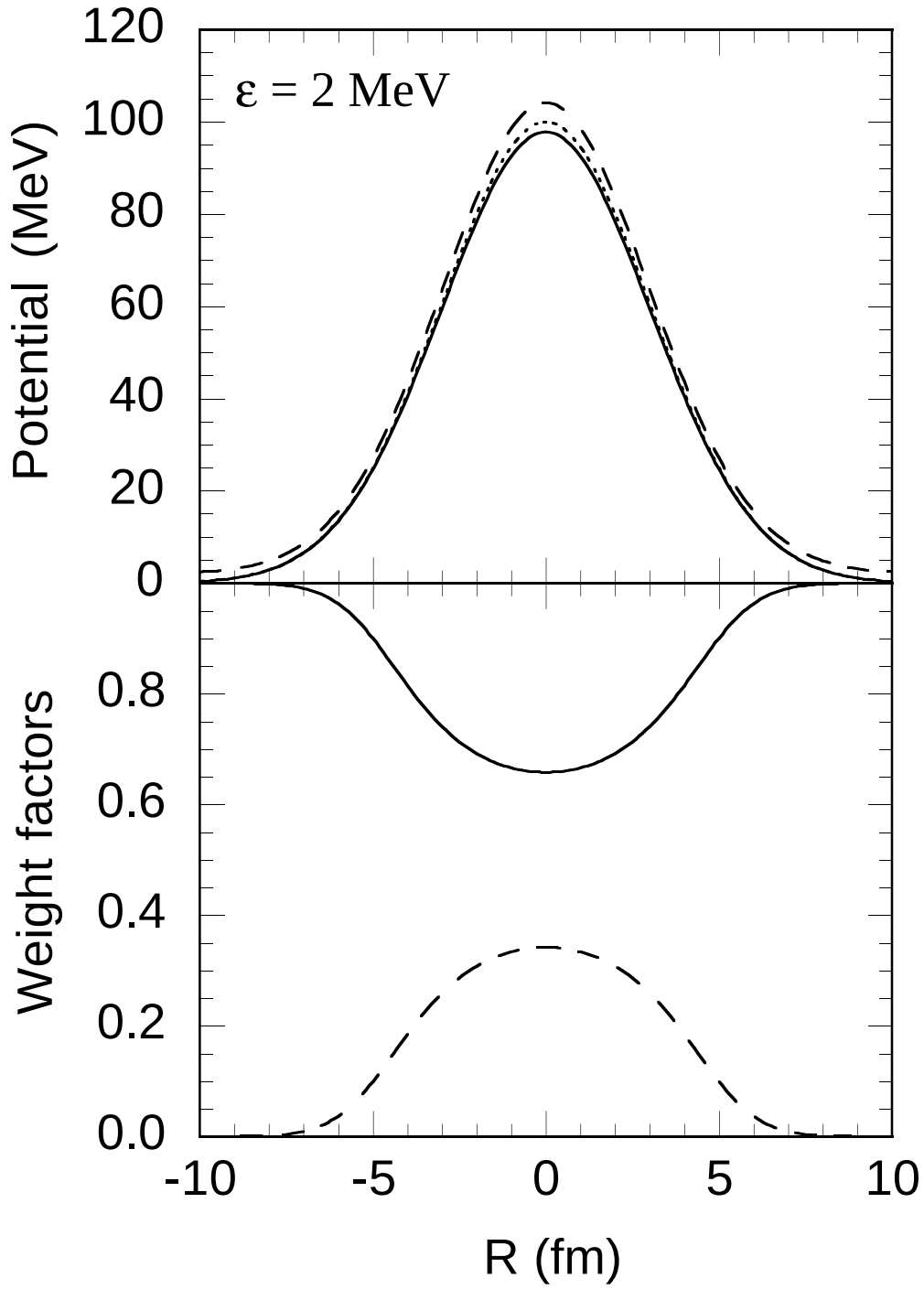
Figure Captions

Fig. 1: The eigen-potentials (the upper panel) and the associated weight factors (the lower panel) as a function of the relative distance R. The solid and the dashed lines in the lower panel are the weight factors for the lower and the higher eigen-potentials, respectively. The dotted line in the upper panel represents the bare potential barrier.
Fig. 2: The penetrability (the first panel) and its first derivative (the second panel) for the two level problem as a function of the incident energy. The third panel shows the optimum weight factors obtained according to eq. (5). The solid and the dashed lines correspond to the weight factors for the lower and the higher potentials, respectively.
Fig. 3: Same as fig. 2, but for different values of the excitation energy of the intrinsic motion which are denoted in the inset. The third panel shows only the weight factors for the lower potential.
Fig. 4: Effects of the coupling to a high excitation energy state. The dotted line is the result without channel coupling, while the solid line is obtained by taking into account the coupling to an excited state whose excitation energy is 8 MeV.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60
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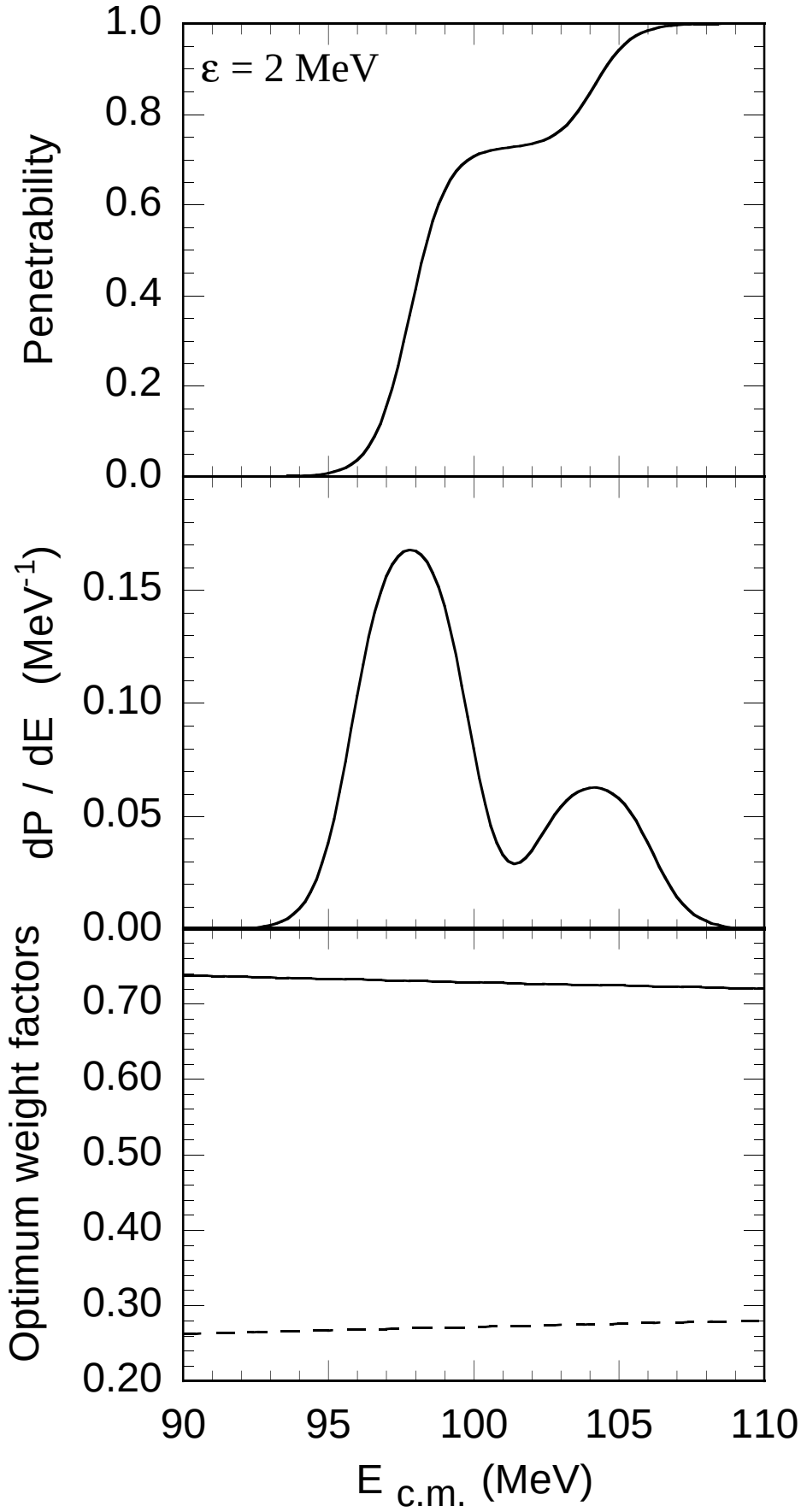
-10
-5
0
5
10
R (fm)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0.10
0.05
0.00
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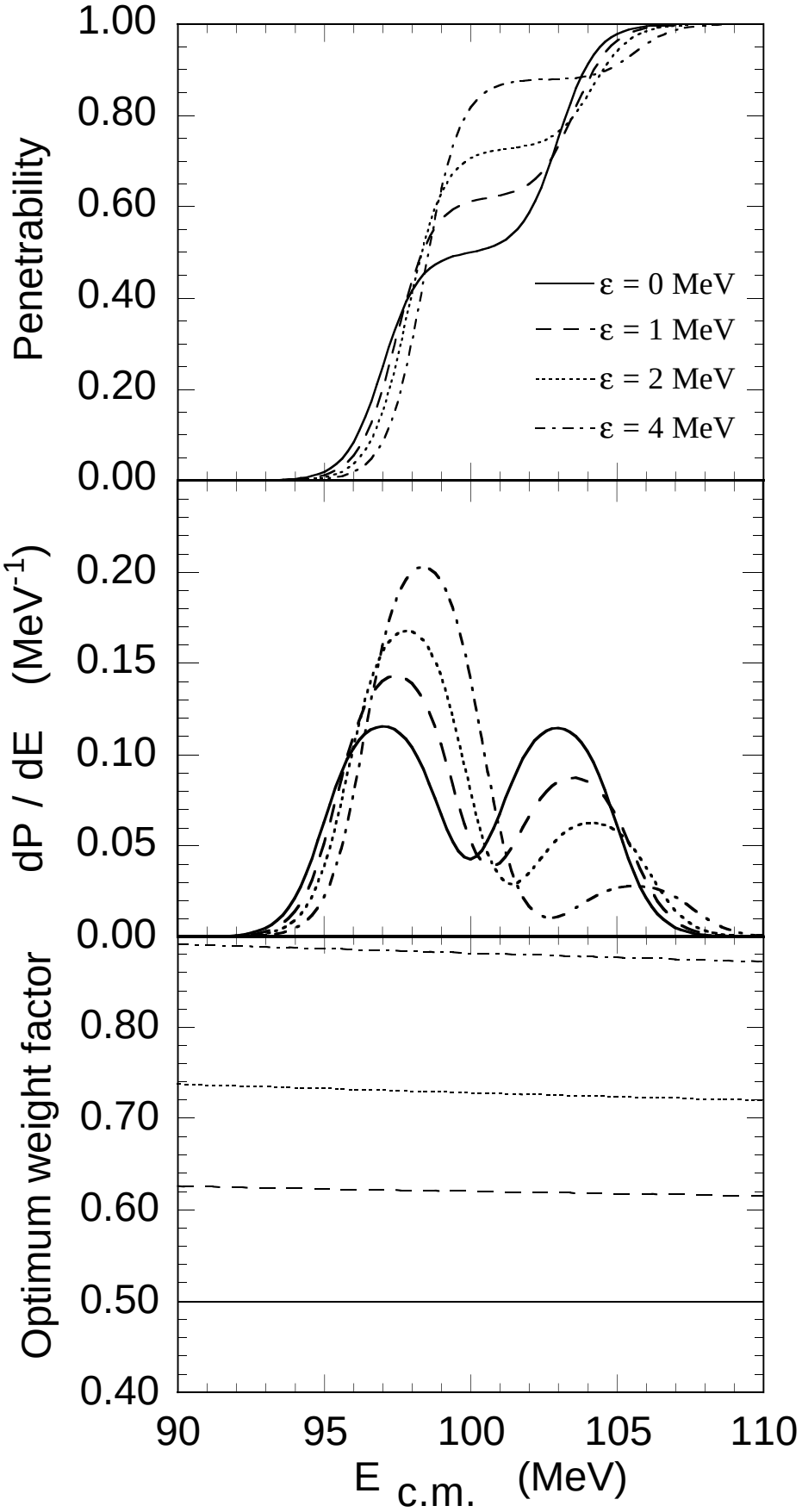
E c.m. (MeV)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ε = 2 MeV
ε = 4 MeV
dP / dE (MeV-1)
0.20
0.15
0.10
0.05
0.00
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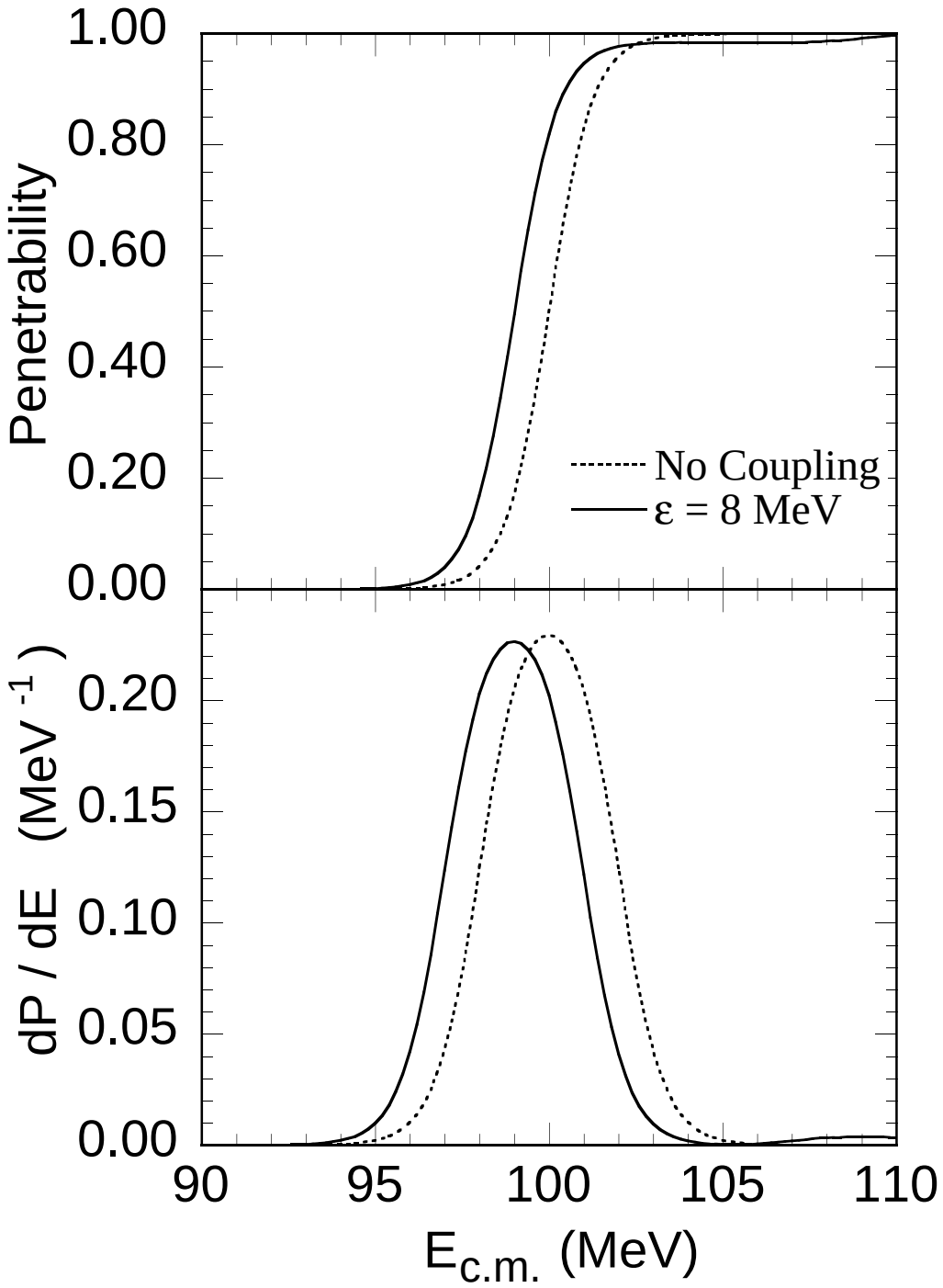
E c.m. (MeV)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Penetrability
No Coupling
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0.05
0.00
dP / dE (MeV -1)
90
95
100
105
110
E c.m. (MeV)